MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2022/2023

DMA5301 – DISCRETE STRUCTURES

(TC1L)

7 JULY 2023
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **3** pages with **5** questions.
2. List of formulae for the logical equivalence laws and rules of inference are in page 3.
3. Answer ALL questions.
4. All necessary working steps must be shown in the Answer Booklet provided.

QUESTION 1 **[7 Marks]**

a) Show that the proposition $(q \wedge p) \rightarrow (r \vee \neg p)$ and $\neg p \vee (q \rightarrow r)$ are logically equivalence by using **logical equivalences' laws**. Present clearly each of your steps and state clearly each of the laws that you have used in your proof. (3 Marks)

b) Use the **Rules of Inference** to prove the following: (4 Marks)

$$(t \vee r) \wedge \neg s \wedge (s \vee \neg t) \wedge (q \rightarrow \neg r) \Rightarrow \neg q$$

QUESTION 2 **[13 Marks]**

a) Let the universe be the set $U = \{1,2,3, \ldots 9,10\}$. Let $A = \{2,4,7,8\}$, $B = \{1,2,5,7,10\}$ and $C = \{x | x \text{ is an odd number}\}$. List the elements of each set below:

i. Set C. (0.5 mark)

ii. $(\overline{B \cap C}) - A$ (1.5 marks)

iii. Power set of A, $P(A)$. (1.5 marks)

b) Let $R = \{(1,1), (1,2), (1,4), (2,2), (2,3), (3,3), (3,4), (4,4)\}$ on set $X = \{1,2,3,4\}$. Is R reflexive, symmetric, antisymmetric and/or transitive? Justify each of your answer. (4 marks)

c) Use a mathematical induction to prove that

$$P_n: 3 + 7 + 11 + \cdots + (4n - 1) = n(2n + 1)$$

is true for all positive integers. (5.5 marks)

QUESTION 3 **[10 Marks]**

a) Given below is the Euclidean Algorithm of Greatest Common Divisor (GCD) for the numbers 180 and 222.

$$\begin{aligned} 222 &= (1)(180) + 42 \\ 180 &= (4)(42) + 12 \\ 42 &= (3)(12) + 6 \\ 12 &= (2)(6) + 0 \end{aligned}$$

i. Find integers s and t such that $180s + 222t = \text{GCD}(180, 222)$. (4 marks)

ii. Find the Least Common Multiple (LCM) of the numbers. (1 mark)

b) In how many ways can 4-letter words be created from the word "SPIDERMAN" with a condition that each resultant 4-letter word is unique? (1.5 marks)

c) In a group of 120 students, 40 students like watermelon, 56 students like apple, and 18 students like both watermelon and apple. Determine the number of students who likes either watermelon or apple. (1.5 marks)

d) Jim is a fencing athlete. Calculate the minimum total training hours for a week if Jim spend at least 9 hours a day for the fencing training. (2 marks)

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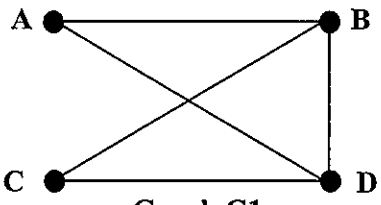
QUESTION 4 **[15 Marks]**

a)



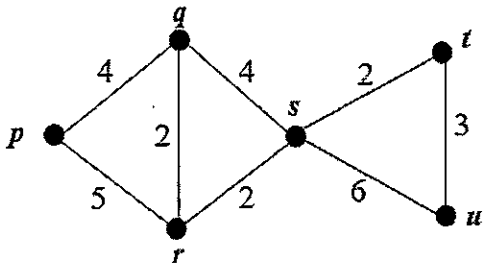
Graph G1

Refer to Graph G1 given above to answer the question:

i. Determine whether Graph G1 has an Euler Cycle or Euler Path. Justify your answer. If the cycle/path exists, indicate the cycle/path. (2.5 marks)

ii. Prove that Graph G1, is a planar graph by indicating the number of faces, vertices and edges. Draw the new planar Graph G1. (2.5 marks)

b) Refer to the weighted Graph G2 given below to answer the question:

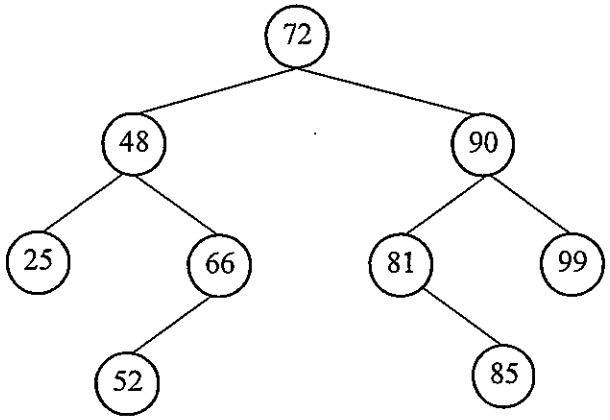


Graph G2

i. Use Dijkstra's algorithm to find the length of a shortest path from vertex ***p*** to ***u***. (4 marks)

ii. Determine the minimum spanning tree using Prim's algorithm. Use vertex ***p*** as the 1st iteration. State the order of the edges that are added to the solution. Then, find the total minimum weighted edges. (4 marks)

c) Refer to the following binary search tree.



List the order in which the vertices are processed using Preorder Traversal and Postorder Traversal. (2 marks)

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QUESTION 5 **[5 Marks]**

Table below denotes the next-state function of a finite-state automaton A having S_0 as initial state and S_2 as the accepting state:

State	Next State	
	a	***b***
S_0	S_0	S_1
S_1	S_2	S_1
S_2	S_2	S_1

a) Construct the state-transition diagram for the A. (2 marks)

b) Find the output string for the given input string and determine whether the given string is accepted or not: ***b b a b a*** (1 mark)

c) Give an example of a 4-characters input string that will be rejected by the automaton. (1 mark)

d) Give an example of a 4-characters input string that will be accepted by the automaton. (1 mark)

FORMULAE

List of Logical Equivalence Laws

Conversion of Implication: $p \rightarrow q \Leftrightarrow \neg p \vee q$

Double Negation: $\neg(\neg p) \Leftrightarrow p$

DeMorgan : (i) $\neg(p \wedge q) \Leftrightarrow (\neg p \vee \neg q)$

(ii) $\neg(p \vee q) \Leftrightarrow (\neg p \wedge \neg q)$

Domination : (i) $p \wedge F \Leftrightarrow F$ (ii) $p \vee T \Leftrightarrow T$

Negation : (i) $p \wedge \neg p \Leftrightarrow F$ (ii) $p \vee \neg p \Leftrightarrow T$

Identity : (i) $p \wedge T \Leftrightarrow p$ (ii) $p \vee F \Leftrightarrow p$

Commutative : (i) $p \wedge q \Leftrightarrow q \wedge p$ (ii) $p \vee q \Leftrightarrow q \vee p$

Idempotent : (i) $p \vee p \Leftrightarrow p$ (ii) $p \wedge p \Leftrightarrow p$

Distributive : (i) $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$

(ii) $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$

Associative : (i) $p \vee (q \vee r) \Leftrightarrow (p \vee q) \vee r \Leftrightarrow p \vee q \vee r$

(ii) $p \wedge (q \wedge r) \Leftrightarrow (p \wedge q) \wedge r \Leftrightarrow p \wedge q \wedge r$

List of Rules of Inference

ADD:	$\dfrac{p}{\therefore p \vee q}$	**SIMP:**	$\dfrac{p \wedge q}{\therefore p}$
Conj:	$\dfrac{p \quad q}{\therefore p \wedge q}$	**Res:**	$\dfrac{p \vee q \quad \neg p \vee r}{\therefore q \vee r}$
MP:	$\dfrac{p \quad p \rightarrow q}{\therefore q}$	**MT:**	$\dfrac{\neg q \quad p \rightarrow q}{\therefore \neg p}$
HS:	$\dfrac{p \rightarrow q \quad q \rightarrow r}{\therefore p \rightarrow r}$	**DS:**	$\dfrac{p \vee q \quad \neg p}{\therefore q}$

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